## Unit 2

## Principle of Projection

## Projection

Projection is defined as a geometrically represented image (visual image or figure) of an object obtained on a surface or plane.

Classification
Projections are basically classified in to two:

1. Perspectiveprojection
2. Parallelprojection

## 1-Perspective Projection

Perspective projection represents objects as perceived by the human eye(s) (refer Figure 1.1). It is a pictorial drawing by the intersection of observer's visual rays (lines of sight) converging on a plane (picture plane). The observer's eye - station point or point of sight - is located at a finite distance from the picture plane (refer Figure 1.1). Depending on the position of the picture plane, the size of the projection may vary.


Figure 1.1 Principle of Perspective Projection

## 2-Parallel Projection

Parallel projection is obtained by assuming the observer at infinite distance from theobject. Hence, the visual rays are considered as parallel to one another. These rays or lines of sight are used to project the object on a standard plane (refer Figure 1.2).The object is projected to a plane by drawing straight lines from each and every point on the object. These lines used for projecting the object are 'projectors'. The plane to which the object is projected is the 'plane of projection'. All projectors are parallel to one another and perpendicular to the plane of projection. The image or view obtained on the plane is the 'projection'.


## Isometric Projection

Isometric projection is a technique where three-dimensional objects are represented in two dimensional drawings. In isometric projection, the three coordinate axes appear equally shortened and the angle between any two of them is 120 degrees (refer Figure 1.3).


Figure 1.3 Isometric projection of a cube

## Perspective Projection

Perspective projection represents objects as perceived by the human eye(s) (refer Figure 1.4).


Figure 1.4 Perspective projection of a cube

## Oblique Projection

Oblique projection is a method to represent 3D objects in 2D drawings in which the projection lines are drawn at $45^{\circ}$ angle to the horizontal.


Figure 1.5 Oblique projection of cube

## Orthographic Projections

In orthographic projection, an object is represented by projecting its views on imaginary orthogonal planes. Any object, irrespective to the dimensions, (1D, 2D or 3D objects) is converted to 2D drawings or projections.

## Reference planes

Principal planes - horizontal plane and vertical plane -are the main reference planes used in orthographic projections.

Profile plane, auxiliary vertical plane, and auxiliary inclined plane are also used as reference planes when two views of the object are not sufficient.
Principal Planes
Horizontal and Vertical planes are the principal planes used in orthographic projections (refer Figure 2.1).

## Horizontal Plane (HP)

A plane of reference which is assumed to be parallel to the plane along the horizon or a plane which is perpendicular to the gravity field at a place. In orthographic projection, there is only one horizontal plane.


Figure 2.1 Principal Planes and Quadrants

## Vertical Plane (VP)

A reference plane which is assumed to be along or parallel to the gravity field. This plane will be perpendicular to the horizontal plane.


Figure 2.2 Auxiliary Vertical Plane

## Auxiliary Inclined Plane (AIP)

Planes perpendicular to VP and inclined to HP are coming under this category. The projection on an AIP is auxiliary top view.

## Profile planes (PP)

Planes perpendicular to both horizontal and vertical planes are profile planes.
Projections on profile planes are known as side views.

## Main Reference Line, x-y

The line of intersection of Horizontal plane (HP) and Vertical plane (VP) is the main reference line, $x-y$ (refer Figure 2.1).

The lines of intersection of auxiliary planes with principal planes are auxiliary reference lines. While drawing auxiliary projections, auxiliary reference lines are used for representing projections.


Figure 2.3 Auxiliary Inclined Plane


Figure 2.4 Profile Plane

## Ground Plane (GP) and Ground Line (g-l)

A plane parallel to HP, assumed to be attached at the bottom most edge of VP is ground plane.
The line of intersection of GP with VP is called as ground line ( $g$ - $l$ ).

## Basic characteristics of planes of projection

Planes are assumed to have enormous area so that any object irrespective to the size can be projected to the plane.

Planes do have negligible thickness, such that they appear as lines while observing along the plane.

Planes are assumed to be transparent, so that irrespective to the quadrantwhere the object situates the observer can view it directly from both front and top.

Planes are not rigidly attached to the other plane(s), such that one plane can be rotated about the line of intersection to make coinciding with the other.


Figure 2.5 Principal Planes, Ground Plane and Quadrants viewing along x-y

## Quadrants

The whole space available is divided in to four quadrants using HP and VP. Viewing along $x-y$, starting from above HP and in front of VP, the numbering of quadrants starts, which is the First quadrant. Moving in anticlockwise direction, above HP and behind VP is the Second quadrant; below HP and behind VP, the Third quadrant, and below HP and in front of VP, the Fourth quadrant (refer Figure 2.1, Figure 2.5)

As GP is attached to the bottom end of VP, there is no space below GP or objects are assumed to be situating either in the First or Second or Third or Fourth quadrant.

## Multi-view Representations

Multi-view representations show more than one standard 2D views of an object. 3D objects are represented by different views or projections on imaginary planes.

In orthographic projections six views (Figure 2.6) of an object can be represented. They are: front view, top view, side view from left (side view-left), side view from right (side view-right), bottom view and back view (refer Figure 2.7). However most common views are top view and front view.


Figure 2.6 Directions of Observation of an Object in the First Quadrant

## First Angle Projection

Projection of any object drawn assuming the object in the first quadrant is First Angle Projection. As per recommendation of Bureau of Indian Standards First angle projection is followed in India. Figure 2.7 shows the symbol for first angleprojection (two views of the frustum of cone).

## Third Angle Projection

Projection of any object drawn assuming the object in the third quadrant is Third Angle Projection. Figure 2.8 shows the symbol of third angle projection.


Figure 2.7 Six Views of the Object in Figure 2.6


Figure 2.7 Symbol of First angle projection


Figur 2.8 Symbol of Third angl projection

## PROJECTION OF POINTS \& LINES

## ABBREVIATIONS

Following abbreviations, symbols and notation will be used in the entire course.

| VP | - Vertical plane |
| :--- | :--- |
| HP | - Horizontal plane |
| XY | - Reference line |
| $\mathrm{x}_{1} \mathrm{y}_{2}, \mathrm{x}_{2} \mathrm{y}_{2}$ | - Auxiliary reference plane |
| HT | - Horizontal trace |
| VT | - Vertical trace |
| A,B,C etc. | - to represent a point/line ends/solid corners in space |
| a,b,c etc. | - to represent top view of a point/line ends/solid corners |
| a', b', c' etc. | - to represent front view of a point/line ends/solid corners |
| AVP | - Auxiliary vertical plane |
| AIP | - Auxiliary inclined plane |
| $\theta$ | - True inclination of a line with HP |
| $\phi$ | - True inclination of a line with VP |
| $\alpha$ | - Apparent inclination of a line with HP |
| $\beta$ | - Apparent inclination of a line with VP |

## INTRODUCTION TO PROJECTION

Projecting the image of an object to the plane of projection is known as projection. The object may be a point, line, plane, solid, machine component or a building. Consider the following illustration to project the image of an object on to a plane.


In engineering drawing practice, two principal planes are used to get the projection of object as shown in figure. They are
(1)

Vertical plane (VP) which is assumed to be placed vertically. The front view of the object is projected onto this plane.
(2) Horizontal plane (HP) which assumed to be placed horizontally. The top view of the object is projected onto this plane.


When an object is assumed to be placed in first quadrant, the projection method followed is called as first angle projection. In this method, the object is placed between the observer and the plane of projection.


When the object is assumed to be placed in third quadrant, the projection method followed is called as third angle projection. In this method, the plane of projection lies between the object and the observer.

## Important Observations

(1) First angle projection is used in USA, whereas Third angle projection is used in European countries.
(2) Bureau of Indian Standards has recommended that only first angle projection method should be used in our country. However, third angle projection system is still accepted
in industrial organizations. Therefore, engineering students must have knowledge of both methods of projections.
(3) An engineer should be comfortable with both, First angle and Third angle projection method
(4) In this entire course, First angle projection method is used.

FIRST ANGLE VS. THIRD ANGLE PROJECTION METHOD

## First Angle Method

1. The object lies In between the observer and the plane of projection. The plane of projection Is always behind the object.
2. The object is assumed to be placed in first quadrant.
3. The front view or the elevation Is always above the top view or the plan.
4. The right hand end view/side view Is drawn to the left and left hand end view Is drawn to the right.
5. The plane of projection may or may not be transparent.
6. It Is represented by the following symbol:


## Third Angle Method

1. Between the observer and the object ore transparent planes of projection. The plane of projection Is always In front of the object.
2. The projections are drawn assuming that Uie object Is situated In third quadrant.
3. The front view Is always below the top view.
4. The right hand end view is drawn to the right and left hand end view Is drawn to the left.
5. The plane of projection Is always transparent.
6. It Is represented by the following symbol :

(1) In drawing practice, capital letters $A, B, C$ etc are used to represent objects in space.
(2) Their top views are represented by small letters a,b,cetc.
(3) The front views are represented by small letters with dashes $\mathrm{a}^{\prime}, \mathrm{b}^{\prime}, \mathrm{c}$ ', etc.
(4) These letters are used to represent a point, ends of a straight line, corners of solid etc.
(5) Actual projections in top and front views are drawn in thick lines, construction lines and projectors are drawn using thin lines.
(6) Top view is also known as plan and front view is known as elevation.

## PROJECTION OF A POINT IN FIRST QUADRANT

Consider a point A placed in the first quadrant. This is at a height h mm above HP, at a distance d mm in front of VP. Its front view a' is projected onto VP and the top view a is projected onto HP.
Now the HP is rotated in the clockwise direction for $90^{\circ}$ and is obtained in vertical position. The projections will be seen as given in Fig.(ii).


It is drawn with reference to XY line. Mark a point a' at a height hmm above XY , and a at a distance d mm below XY. The projector joining a' and a is always perpendicular to XY.

## PROJECTION OF POINT IN THIRD QUADRANT

Consider a point C placed in the third quadrant. The point is at a height $\mathrm{h} m \mathrm{~mm}$ below HP and a distance d mm behind VP. Its front view c' is projected onto VP and the top view c is projected onto HP.


Now the HP is rotated in the clockwise direction for $90^{\circ}$ and is obtained in vertical position. The projections will seen as given in Fig.(ii). It is drawn with reference to XY line. Mark a point c ' at a height h mm below XY and c at a distance d mm above XY .

## EXAMPLE

A point A is 20 mm above $H P$ and 30 mm in front of VP. Draw its projection.


## To draw the projections

Draw the reference line XY. Mark a point a' at a distance of 20 mm above XY. Through this point draw a perpendicular line to XY and mark the top view a at a distance of 300 mm below XY.

A point $C$ is 20 mm below HP and 30 mm behind VP. Draw its projection.


(ii)

## To draw the projections

Draw the reference line XY. Mark a point c' at a distance of 20 mm below XY. Through this point draw a perpendicular line to XY and mark the top view c at a distance of 30 mm above XY.

## PROJECTION OF STRAIGHT LINES

A straight line is the shortest route to join any two given points. It is a one-dimensional object having only length (1).

The projection of straight line are obtained by joining the top and front views of the respective end points of the line. The actual length of the straight line is known as true length (TL).

## Example

Explain true length concept with an example.

## Solution

Hold a pencil in your hand and open a note-book in such a way that some pages are on the table and some pages are held perpendicular to the table top. Consider these as two planes HP and VP.

Put one end of the pencil on the HP and let it make some angle with HP. Make the pencil parallel to VP (imagine the pencil to be a line).
Under the condition if the line is projected on HP and VP, its projections will be as shown in following figure.


The elevation of the line $\mathrm{a}^{\prime} \mathrm{b}^{\prime}$ will be true length, the angle $\theta$ will also be true angle but plan ab will be shorter than true length. The length ab is a function of angle $\theta$. If $\theta$ is increased, $a b$ will be reduced further.

Now consider the pencil parallel to HP and making some angle $\varphi$ with VP. Its projections will be as shown in following figure.


The length of plan is True length TL, the angle made by this plan with VP, i.e. with XY line is true angle $\varphi$, and the length of elevation is a function of angle $\varphi$. Elevation is parallel to XY (in second case, the line is given parallel to HP, while in this case the elevation is a line parallel to XY and the plan of the line is TL, the angle made $\varphi$ is true angle).

From the example, we see that in first case when the line was parallel to VP, the elevation of the line is TL, the angle made, i.e. $\theta$ o, is also true angle and the plan is a line parallel to $\mathrm{X}-\mathrm{Y}$.

Hence, learn a rule that if the projections of the line are given and if one of the views (plan or elevation) is parallel to XY, the other view MUST BE true length and angle made by this TL will be true angle. Its reverse is also true, i.e. from among the views of a line, if one view is TL, the other view MUST BE parallel to X-Y.

A straight line is placed with reference to the planes of projections in the following positions.

1. Line perpendicular to $H P$ and parallel to $V P$.
2. Line perpendicular to $V P$ and parallel to $H P$.
3. Line parallel to both $H P$ and $V P$.
4. Line inclined to $H P$ and parallel to $V P$.
5. Line inclined to $V P$ and parallel to $H P$.
6. Line inclined to both $H P$ and $V P$.

In first angle projection method, the line is assumed to be placed in the first quadrant. The projections of the line in the above mentioned positions are discusses below.


## Projection of a Line kept Perpendicular to HP and Parallel to VP

Consider a straight line AB kept perpendicular to HP and parallel to VP Fig.(i). Its front view is projected onto VP which is s line having true length. The top view is projected onto HP which is a point one end $b$ of which is visible while the other end a is invisible and is enclosed within ().
Now the HP is rotated in the clockwise direction through $90^{\circ}$ and is obtained in the vertical position. The projections obtained are seen as given in Fig. It is drawn with reference to the XY line as follows.

1. Draw the XY line.
2. Draw the front view a'b'. which is a line perpendicular to $X Y$ and having true length (TL).
3. Projected the top view $a b$. The end $b$ is visible and the invisible end $a$ is marked inside ().

## Projection of a Line kept Perpendicular to VP and Parallel to HP


(ii)

Consider a straight line AB kept perpendicular to VP and parallel to HP Fig.(i). Its top view is projected onto HP which is a line having true length (TL). The front view is projected onto VP which is point, the end $b$ ' of which is visible and the other end $a^{\prime}$ is invisible which is shown enclosed in( ).

Now the HP is rotated in the clockwise direction through $90^{\circ}$ and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows.

1. Draw the XY line.
2. Draw the top view $a b$, a line perpendicular to $X Y$ and having true length (TL).
3. Projected the front view $a^{\prime} b$ '. The end $b$ ' is visible ad the invisible end $a$ ' is marked inside ().

## Projection of a Line kept Parallel to Both HP and VP


(i)

(ii)

Consider a straight line AB kept parallel to both HP and VP Fig.(i). Its front view is projected onto VP which is a line having true length (TL). The top view is projected onto HP which is also a line having true length.

Now the HP is rotated in the clockwise direction through an angle of $90^{\circ}$ and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with references to the XY line as follows:

1. Draw the XY line.
2. Draw the front view $a^{\prime} b^{\prime}$, a line parallel to $X Y$ and having true length (TL)
3. Project the top view ab which is also a line parallel to XY having true length (TL).

## Projection of a Line kept inclined to HP and Parallel to VP

Consider a straight line AB kept inclined to HP and parallel to VP Fig.(i). Its front view is projected onto VP which is an inclined line at an angle $\theta$ to XY and having true length (TL). The top view is projected onto HP which is also a line but smaller than the true length and parallel to XY. The inclination of the line with HP is always represented by the symbol $\theta$.

(i)

(ii)

Now the HP is rotated in the clockwise direction through $90^{\circ}$ and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows:

1. Draw the XY line.
2. Draw the front view a'b', a line inclined at an angle $\theta$ to $X Y$ having true length (TL).
3. Project the top view ab which is also a line parallel to XY and smaller than true length.

## Projection of a Line kept Inclined to VP and Parallel to HP

Consider a straight line AB kept inclined to VP and parallel to HP Fig.(i). Its top view is projected onto HP which is a line inclined at an angle $\phi$ to XY and having true length (TL). The front view is projected onto VP which is also a line but smaller than true length and is parallel to XY. The inclination of the line with VP is always represented by the symbol $\phi$.

(i)

(ii)

Now the HP is rotated in the clockwise direction through an angle of $90^{\circ}$ and is obtained in the vertical position. The projections obtained are seen as given in Fig.(ii). It is drawn with reference to the XY line as follows:

1. Draw the XY line
2. Draw the top view ab , a line inclined at an angle $\phi$ to $X Y$ and having true length (TL).
3. Project the front view a'b', which is also a line parallel to XY but smaller than true length.

## TRACE OF A LINE

The point of intersection or meeting of a line with the reference plane, extended if necessary, is known as the trace of a line. The point of intersection of a line with the HP is known as the horizontal trace, represented by HT and that with the VP is known as the vertical trace, represented by VT. No trace is obtained when a line is kept parallel to a reference plane.
If the line is given parallel to a plane, it will never intersect that plane and, therefore, no trace of the line on that plane.

If the line is given parallel to VP and inclined to HP, only HT will be obtained and no VT.
If the line is given parallel to HP and inclined to VP, only VT will be obtained and no HT.
If the line is given parallel to both the planes, neither HT nor VT will be obtained.

## Rule For obtaining HT of an inclined line $A B$

For obtaining HT of an inclined line $A B$, inclined to HP, extend the elevation $a^{\prime} b^{\prime}$ towards $X$ $Y$ line till it intersects $X-Y$ (In some problems, depending upon the position of the given line, the elevation might intersect $X-Y$ line in natural course and it need not be extended. This may happen when one end of the line is above HP and the other end below HP. In this also the point of intersection of elevation with $X-Y$ will be $h^{\prime}$.) The point of intersection of the extended elevation with $X-Y$ is $h^{\prime}$.


Now from $h^{\prime}$, draw a line perpendicular to $X-Y$. Then extend the plan $a b$ in its own direction, till it intersects the perpendicular from $h^{\prime}$.

## Rule For obtaining VT of an inclined line $A B$

The point of intersection of extended plan $a b$ (in some cases the plan need not be extended but the perpendicular from $h^{\prime}$ will intersect the plan $a b$ ) and the perpendicular from $h^{\prime}$ will be HT.

For obtaining the VT the procedure is reversed
(a) Extend the plan towards $\mathrm{X}-\mathrm{Y}$ line till it intersects $\mathrm{X}-\mathrm{Y}$. The point of intersection is v .
(b) Draw a line perpendicular to $\mathrm{X}-\mathrm{Y}$ from h .
(c) Extend the elevation in its own direction, so that it intersects the perpendicular drawn from h .

(d) The point of intersection is VT. (In some cases, the elevation need not be extended and the perpendicular from h will intersect the elevation. This point of intersection is VT)

## EXAMPLE

A line $A B 60 \mathrm{~mm}$ long has its end 20 mm above $H P$ and 30 mm in front of VP. The line is kept perpendicular to HP and parallel to VP. Draw its projections. Also mark the traces.

## SOLUTION

Assume that end A of the line is nearer to HP. The front view a'b' is a line having true length. The top view is a point, the end $b$ of which is visible and $a$ is invisible which is enclosed in (). The line is extended to meet HP to obtained the horizontal trace (HT). No vertical trace (VT) is obtained because the line is kept parallel to VP.


The projections obtained are drawn with reference to XY line as shown in Fig.12(ii).
(You should not drawn diagram (i) in the examination. This diagram is given for explanation only.)

1. Mark the projections of the end $A$ by considering it as a point. Its front view a' is 20 mm above XY and the top view a is 30 mm below XY.
2. The front view of the line $a^{\prime} b$ ' is obtained by drawing a line perpendicular to $X Y$ from a' and having a length of 60 mm .
3. Top view of the line is obtained by projecting the other end $b$ which coincides with $a$. The invisible end a is enclosed in ().
4. The horizontal trace (HT) is marked coinciding with the top view of the line. NO vertical trace (VT) is obtained.

## EXAMPLE

A line $A B 60 \mathrm{~mm}$ long has its end $A 20 \mathrm{~mm}$ above $H P$ and 30 mm in front of $V P$. The line is inclined at $40^{\circ}$ to $V P$ and parallel to $H P$. Draw its projections. Also mark the traces.

## SOLUTION

The top view ab is a line inclined at an angle of $40^{\circ}$ to XY and having true length. Its front view $a^{\prime} b^{\prime}$ is parallel to $X Y$ and smaller than true length.

The line is extended to meet VP to obtained the vertical trace (VT). No horizontal trace (HT) is obtained because the line is kept parallel to HP.
The projections obtained are drawn with reference to XY line as shown in Fig.(ii).


1. Mark the projections of end $A$ by considering it as a point. Its front view a' is 20 mm above XY and top view a is 30 mm below XY.
2. The top view of the line ab is obtained by drawing a line inclined at an angle $40^{\circ}$ to XY from a and having a length of 60 mm .
3. The front view of the line $a^{\prime} b^{\prime}$ is obtained by drawing a line parallel to XY from a' and drawing a vertical line (projector) from $b$. It is parallel to XY and smaller than the true length.
4. To mark the vertical trace (VT) the top view of the line ab is extended to intersect with XY line at $v$. Then by drawing a vertical line from $v$ and a horizontal line from a'b', the VT is located. No horizontal trace (HT) is obtained.

## EXAMPLE

A line $A B 55 \mathrm{~mm}$ long has its end 25 mm in front of $V P$ and in $H P$. The line is inclined at $45^{\circ}$ to VP. Draw its projections.

## SOLUTION

The projections obtained are drawn with reference to the XY line as shown in Fig.(ii).

1. Mark the projections of the end A by considering it as a point. Its top view A is 25 mm below XY and front view $a^{\prime}$ is on the XY line.
2. The top view of the line $a b$ is obtained by drawing a line inclined at $45^{\circ}$ to $X Y$ from a and having a length of 55 mm .
3. The front view of the line $a^{\prime} b^{\prime}$ is obtained by drawing a line on $X Y$ from $a^{\prime}$ and drawing a vertical line from $b$. It is smaller than the true length.

(i)

(ii)

## When a line is inclined to one plane and parallel to the other plane.

## (a) When a line is inclined to HP and parallel to VP

The projections are usually obtained as follows. There are three variables namely TL, $\theta$ and TV marked in the drawing. In a problem usually any two variable values will be given and the third variable value can be obtained graphically by completing the drawing as mentioned below. Draw the projections a' and a of the given end A.
(i) When TL and $\theta$ are given. Draw the front view a'b' from a' using TL and $\theta$. Top view (TV) ab is projected and obtained by drawing a line parallel to the XY line and a vertical line (projector) from b'.
(ii) When TL and TV are given. Draw the top view ab using TV parallel to XY line. Draw the vertical line (projector) from $b$. Using TL as radius and a' as centre, mark a point in the vertical line to get b'. Join a' and b' to complete the front view $a^{\prime} b^{\prime}$ of the line. The inclination of a'b' with XY is measured to get $\theta$.
(iii) When TV ad $\theta$ are given. Draw the top view using the length of TV parallel to the XY line. Draw the vertical line (projector) from $b$. Using the angle $\theta$, draw a line which intersects the projector $a t b^{\prime}$. Join $a^{\prime}$ and $b$ ' to front view $a$ ' $b$ ' of the line. The length of $a^{\prime} b$ ' is measured to get TL.


## (b) When a line is inclined to VP and parallel to HP

The projections are usually obtained as follows.


There are three variables namely TL, $\phi$ and FV marked in the drawing. In a problem usually any two variable values will be given the third variable value can be obtained graphically by completing the drawing as mentioned below. Draw the projections a' and a of the given end A.
(i) When TL and $\phi$ are given. Draw the top view ab using TL and $\phi$. The front view ( FV ) a'b' is projected and obtained by drawing a line parallel to XY and a vertical line (projector) from $b$.
(ii) When TL and FV are given. draw the front view a'b' using FV parallel to the XY line. Draw the vertical line (projector) from $b^{\prime}$. Using TL as radius and a as centre, mark a point in the vertical line to get $b$. Join $a$ and $b$ to top view $a b$ of the line. The inclination of $a b$ with XY is measured to get $\phi$.
(iii) When FV and $\phi$ are given. Draw the front view a'b' using FV parallel to the XY line. Draw the vertical line (projector) from b'. Using the angle $\phi$, draw a line
which intersects with the projector at $b$. Join $a$ and $b$ to complete the top view $a b$ of the line. The length of $a b$ is measured to get TL.

EXAMPLE
A line AB 70 mm long has its end A 15 mm above HP and 25 mm in front of VP. Its top view (plan) has a length of 40 mm . Draw its projections and find the inclination of the line with HP.

## SOLUTION

The projections of the line are drawn with reference to the XY lien as follows:

1. Mark the projections of end A by considering it as a point. Its front view a' is 15 mm above XY and top view a is 25 mm below XY.
2. The top view of the line $a b$ is drawn parallel to $X Y$ to the given length of 40 mm .
3. Draw a vertical line (projector) from $b$.
4. Using true length 70 mm as radius and a' as centre, mark a point in the vertical line to get b'
5. The inclination of a'b' with $X Y$ is measured to get $\theta$.


## EXAMPLE

A line AB has its end 30 mm above HP and 20 mm front of VP. Its plan has a length of 45 mm . The line is inclined at $50^{\circ}$ to HP and parallel to VP. Draw its projections and find the true length of the line.


The projections of the line are drawn with reference to the XY line as follows.

1. Mark the projections of end $A$ by considering it as a point. Its front view a' is 30 mm above XY and top view a is 20 mm below XY.
2. The top view a is 20 mm below XY .
3. Draw a vertical line (projector) from $b$.
4. Using the angle $50^{\circ}$, draw a line from a' to get the front view a'b' of the line.
5. The length of $a$ ' $b$ ' is measured to get the true length of the line.

## EXAMPLE

A line $A B 55 \mathrm{~mm}$ long is in $H P$ and inclined to $V P$. The end $A$ is 20 mm in front of $V P$. The length of front view is 35 mm . Draw the projections of the line and also find the inclination of the line with $V P$.

## SOLUTION

The projections of the line are drawn with reference to the XY line as follows:

1. Mark the projections of end $A$ by considering it as a point. Its front view a' is on $X Y$ and top view a is 20 mm below XY.
2. The front view a'b' of the line is drawn on $X Y$ for a length of 35 mm .
3. Draw a vertical line (projector) from b'.
4. Using true length 55 mm as radius and a as centre, mark a point in the vertical line to get $b$. Join $a$ and $b$ to complete the top view $a b$ of the line.
5. The inclination of ab with XY is measured to get $\phi$.


## Projection of a Line kept Inclined to Both HP and VP

When a line is placed inclined to both HP and VP, its projections obtained in top and front views are smaller than the true length of the line and inclined to the XY line. So it is impossible to project and draw the top or front view of the line directly. Any one of the following methods may be used to draw the projections.

1. Rotating line method
2. Rotating trapezoidal plane method
3. Auxiliary plane method

## 1. Rotating Line Method

Consider a line AB is placed inclined at $\theta$ to HP and $\phi$ to VP. Draw its projections assuming that the line is placed in the first quadrant. The following steps are to find the top view (plan) and front view (elevation) lengths and then, they are rotated to the required position to represent the projections of the line in the given position.

Mark the projections of the end A by considering it as a point. Its front view a' will be obtained above XY and top view a will be obtained below the XY line.
Step 1: Assume that the line is kept inclined to HP and a re parallel to VP. Draw the front view a' $b_{1}^{\prime}$, it is a line inclined at $\theta$ to XY ad having true length (TL). Project and get the top view $\mathrm{ab}_{1}$ length which is parallel to XY line. Then this will be rotated to the required position. [Figure.(i)].

Step2: Assume the line is kept inclined to VP and parallel to HP. Draw the top view $\mathrm{ab}_{2}$, it is a line inclined at $\phi$ to XY and having true length (TL). Project and get the front view $\mathrm{ab}_{2}^{\prime}$ length which is parallel to XY line. Then this will be rotated to the required position. [Figure (ii)].

(i)

(ii)

Step3: Draw the locus of the other end B of the line in top and front views. Draw the locus of the front view b' as a line passing through $\mathrm{b}_{1}^{\prime}$ and parallel to XY line. Draw the locus of the top view $b$ as a line passing through $\mathrm{b}_{2}$ and parallel to XY line [Fig.(iii)]. Note that step 1 and step 2 are shown together.

Step4: Rotate the top view $a b_{1}$ and front view $a$ ' $b_{2}$ to the required position. Take a as centre, top view length $\mathrm{ab}_{1}$ as radius, draw an arc to intersect with the locus of b at b . Join a and $b$ to get the top view $a b$ of the line in required position. Taking $a$ ' as centre, front view a' $b_{2}^{\prime}$ as radius, draw an arc to intersect the locus of $b$ ' at $b$ '. Join $a$ ' and $b$ ' to get the front view a'b' of the line in required position [Fig.(iv)].

(i)

(ii)

Check the drawing obtained, by drawing the projector for the end $B$ by joining $b$ ' and $b$ which is a line always perpendicular to XY line.

## EXAMPLE

A line $A B 80$ mm long has its end $A 20 \mathrm{~mm}$ above $H P$ and 25 mm in front of $V P$. The line is inclined at $45^{\circ}$ to $H P$ and $35^{\circ}$ to VP. Draw its projections.


Mark the projections of end A by considering it as a point. Its front view a' is 20 mm above XY and top view a is 25 mm below the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view $\mathrm{a}^{\prime} \mathrm{b}$ ', a line inclined at $45^{\circ}$ to Xy line and having a length of 80 mm . Project and get the top view $\mathrm{ab}_{1}$ length which is parallel to XY line.
2. Assume that the line kept inclined to VP and parallel to HP. Draw the top view $\mathrm{ab}_{2}$, a line inclined at $35^{\circ}$ to $X Y$ line and having a length of 80 mm . Project and get the front view $\mathrm{a}^{\prime} \mathrm{b}_{2}$ length which is also parallel to XY line.
3. Draw the locus of the other end $B$ of the line in top and front views. Draw the locus of $b^{\prime}$ which is a line passing through $b_{1}^{\prime}$ and parallel to XY line. Also draw the locus of $b$ which is a line passing through $b_{2}$ and parallel to XY line.
4. Rotate the top view $a b_{1}$ and front view $a^{\prime} b_{2}$ to the required position. Take a as centre, top view length $a b_{1}$ as radius, draw an arc to intersect the locus of $b$ at $b$. Join and $b$ to get the top view $a b$ of the line. Take $a^{\prime}$ as centre, front view length $\quad a^{\prime} b_{2}^{\prime}$ as radius, draw an arc to intersect the locus of b' at b'. Join a' and b' to get the front view a'b' of the line.
5. Check the result obtained by drawing the projector joining $b$ ' and $b$ which should be $a$ vertical line.

A line $A B 70 \mathrm{~mm}$ long, has its end $A 35 \mathrm{~mm}$ above $H P$ and 30 mm in front of $V P$. The top view and front view has a length of 45 mm and 60 mm respectively. Draw its projections.

## SOLUTION



## ANSWERS

$\theta=50^{\circ}$
$\phi=31^{\circ}$

Mark the projections of the end A by considering it as a point. Its front view a' is 35 mm above $X Y$ and top view a is 30 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. In this case, considering the given data the top view $\mathrm{ab}_{1}$ can be drawn parallel to XY and having a length of 45 mm . Draw a vertical (projector) through $b_{1}$. Using true length 70 mm as radius and a' as centre, draw an arc to intersect the vertical line through $b_{1}$ to get $b_{1}$ which represents the true length of the line. The inclination of $a^{\prime} b$ to XY is the inclination of the line with $\operatorname{HP}(\theta)$.
2. Assume that the line is kept inclined to VP and parallel to HP. In this case, considering the given data, the front view $a^{\prime} b_{2}$ can be drawn parallel to XY and having a length of 60 mm . Draw vertical line (projector) through $\mathrm{b}_{2}$. Using true length 70 mm as radius and a as centre, draw an arc to intersect the vertical line
through $b_{2}$ to get $b_{2}$. Join $a$ and $b_{2}$ which represents the true length of the line. The inclination $\mathrm{ab}_{2}$ to XY is the inclination of the line with $\operatorname{VP}(\phi)$.
3. Draw the locus of the other end $B$ of the line in top and front views. Draw the locus of b' which is a line passing trough $\mathrm{b}^{\prime}$ and parallel to XY line. Also draw the locus of b which is a line passing through $b_{2}$ and parallel to XY line.
4. Rotate the top view $\mathrm{ab}_{1}$ and front view $\mathrm{a}^{\prime} \mathrm{b}_{2}$ to the required position. Take a as centre, top view $a b_{1}$ as radius, draw an arc to intersect with the locus of $b$ at $b$. Join $a$ and $b$ to get the top view $a b$ of the line. Take $a^{\prime}$ as centre, front view $a^{\prime} b_{2}^{\prime}$ as radius, draw an arc to intersect the locus of $b^{\prime}$ at $b^{\prime}$. Join $a^{\prime}$ and $b^{\prime}$ to get the front view $a^{\prime} b^{\prime}$ of the line.
5. Check the result obtained by drawing the projector joining $b$ ' and $b$ which should be $a$ vertical line.

## Example

A line $A B 75 \mathrm{~mm}$ long has its end $A$ in both $H P$ and $V P$. The line is kept inclined at $45^{\circ}$ to $H P$ and $30^{\circ}$ to $V P$.

## Solution

Mark the projections of end A by considering it as a point. Its front view a' and top view "a" are marked as a common point on the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view $a^{\prime} b_{1}{ }^{\prime}$ which is inclined at $45^{0}$ to $X Y$ and has a length of 75 mm . The top view length $\mathrm{ab}_{1}$ is projected and obtained on XY line.
2. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view $\mathrm{ab}_{2}$ which is inclined at $30^{\circ}$ to $X Y$ and has a length of 75 mm . The front view length $a^{\prime} b_{2}{ }^{\prime}$ is projected and obtained on $x Y$ line.
3. Draw the locus of $b^{\prime}$, passing through $b_{1}{ }^{\prime}$ and parallel to XY line. Also draw the locus of $b$ passing through $b_{2}$ and parallel to XY line.
4. Rotate the top view $a b_{1}$ by taking " $a$ " as centre, $a b_{1}$ as radius to get the intersection point $b$ with the locus of $b$. Join $a$ and $b$ to complete the top view $a b$ of the line. Rotate the front view $\mathrm{a}^{\prime} \mathrm{b}_{2}{ }^{\prime}$ by taking $\mathrm{a}^{\prime}$ as centre, $\mathrm{a}^{\prime} \mathrm{b}_{2}{ }^{\prime}$ as radius to get the intersection point $b^{\prime}$ with the locus of $b^{\prime}$. Join a' and b' to get the front view a'b' of the line.
5. Check the result obtained by drawing the projector joining b' and $b$ which should be $a$ vertical line.


## EXAMPLE

A line $A B 85 \mathrm{~mm}$ long has its end $A 25 \mathrm{~mm}$ away from both the reference planes and is in the first quadrant. The line is inclined at $50^{\circ}$ to HP and $30^{\circ}$ to $V P$. Draw its projections and mark the traces of the line.

## SOLUTION

Mark the projections of end A by considering it as a point. Its front view a' is 25 mm above XY and top view a is 25 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel $t$ VP. Draw the front view $a^{\prime} \mathrm{q}^{\prime}$ which is inclined at $50^{\circ}$ to XY line and has a length of 85 mm . The top view length $a b_{1}$ is projected and obtained parallel to XY line.
2. Assume that the line is kept inclined VP and parallel to HP. Draw the top view $\mathrm{ab}_{2}$ which is inclined at $30^{\circ}$ to XY line and has a length of 85 mm . The top length $\mathrm{ab}_{1}$ is projected and obtained parallel to XY line.
3. Draw the locus of $b^{\prime}$, passing through $b_{1}^{\prime}$ and parallel to XY line. Also draw the locus of $b$, passing through $b_{2}$ and parallel to XY line.
4. Rotate the top view $\mathrm{ab}_{1}$ to the required position by taking a as a centre, $\mathrm{ab}_{1}$ as radius to get the intersection point $b$ with the locus of $b$. Join $a$ and $b$ to complete the top view ab of the line.
5. Rotate the front view $a^{\prime} b_{2}^{\prime}$ by taking $a^{\prime}$ as centre, $a^{\prime} b_{2}^{\prime}$ as radius to get the intersection point $b$ ' with the locus of $b$ '. Join $a$ ' and $b$ ' to get the front view $a$ ' $b$ ' of the line.
6. Check the result obtained by drawing the projector joining $b$ ' and $b$ which should be $a$ vertical line.


## To mark the traces

1. Extended the front view a'b' to get the intersection point h' with XY line.
2. Produce the top view ab to get the intersection point $v$ with XY line.
3. Draw the vertical line from h' to intersect with the top view to get the horizontal trace (HT) of the line.
4. Draw another vertical line from $v$, to intersect with the front view to get the vertical trace (VT) of the line.

## EXAMPLE

One end $A$ of a line $A B, 75 \mathrm{~mm}$ long is 20 mm above $H P$ and 25 mm in front of $V P$. The line is inclined at $30^{\circ}$ to $H P$ and the top view makes $45^{\circ}$ with VP. Draw the projections of the line and the find the true inclinations with the vertical plane.

Mark the projections of end of A. Its front view a' is 20 mm above XY and top view a is 25 mm below XY line.


1. Assume that the line kept inclined to HP and parallel to VP. Draw the front view a' $b_{1}$ inclined at $30^{\circ}$ to XY and having a length of 75 mm . The top view length $\mathrm{ab}_{1}$ is projected and obtained parallel to the XY line.
2. From top view a draw a line inclined at $45^{\circ}$ to $X Y$, where the top view $b$ of the line will be obtained. Rotate the top view length $a b_{1}$ with the centre a to get the intersection point $b$. Join $a$ and $b$ to get the top view $a b$ of the line.
3. Draw the locus of $b^{\prime}$, passing through $b_{1}^{\prime}$ and parallel to XY line. Also draw the locus of $b$, passing through $b$ and parallel to XY line.
4. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view by drawing an arc with a as centre and radius 75 mm to intersect the locus of $b$ at a point $b_{2}$. Join $a$ and $b_{2}$ to get the inclination with $\operatorname{VP}(\phi)$. The front view length $a^{\prime} b_{2}$ is projected and obtained parallel to XY line.
5. Rotate the front view a' $b_{2}^{\prime}$ as radius to get the intersection point $b^{\prime}$ with the locus of $b^{\prime}$. Join $a^{\prime}$ and b' to get the front view a'b' of the line.
6. Check the result by drawing the projector joining $b$ ' and $b$ which should be a vertical line.

## Projections of a Line when One End of the Line is in HP and the other in VP

This is considered a special case, when only one condition is given for both ends, one end in HP and another end in VP.

Consider a line AB having its end A is HP , (its position from VP not given) and another end B in VP (its position from HP is not given).

The drawing procedure for this line is the same as in the previous problems, but steps involved in drawing the projections are drawn separately to get the final projections.

The vertical trace (VT) will coincide with the end touching the VP and the horizontal trace (HT) will coincide with the end touching the HP.

EXAMPLE
A straight line $A B$ is inclined $45^{\circ}$ to $H P$ and $30^{\circ}$ to $V P$. The point $A$ is in $H P$ and the point $B$ is $V P$. The length of the straight line is 70 mm . Draw the projections of the straight line $A B$. Also mark the traces.

## SOLUTION

1. Mark the front view a' of the end A on XY line arbitrarily. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view a' $b_{1}$ ' of the line which is inclined at $45^{\circ}$ to $X Y$ and has a length of 70 mm . The top view $\mathrm{ab}_{1}$ length is projected and obtained on the XY line.
2. Mark the top view $b$ of the end $B$ on XY line arbitrarily. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view $\mathrm{ba}_{2}$ of the line which is inclined at $30^{\circ}$ to XY and has a length of 70 mm . The front view $\mathrm{ba}_{2}{ }_{2}$ length is projected and obtained on XY line.
3. Draw the locus of $b$ ' passing through $b_{1}$ and parallel to XY line. Also draw the locus of a passing through $\mathrm{a}_{2}$ and parallel to XY line.
4. Mark the front view a' on XY line arbitrarily in another position to get the projections. Considering a' as centre, front view length ba ${ }_{2}$ as radius, draw an arc to get b' in the locus of b'. Join a' and b' to get the front view a'b' of the line. Draw the projector passing through $b^{\prime}$ to mark the top view $b$ on XY line. Considering $b$ as centre, top view length $a^{\prime} b_{1}^{\prime}$ as radius, draw an arc to get a in the locus of a. Join a and $b$ to get the top view $a b$ of the line.
5. Check the result obtained by drawing the projector joining a' and a which should be a vertical line.

The end A is in HP, so the horizontal trace (HT) is marked coinciding with the top view a of the line. The end B is in VP, so the vertical trace (VT) is marked coinciding with the front view $b$ ' of the line.


## EXAMPLE

A line $A B 85$ mm long its end A 25 mm above the $H P$ and 20 mm in front of $V P$. The end $B$ is 60 mm above HP and 50 mm in front of VP. Draw the projections and find its inclination with $H P$ and $V P$.

## SOLUTION

Mark the projections of end A. Its front view a' is 25 mm above XY and top view a is 20 mm below the XY line.

1. Draw the locus of the other end B in front and top views. Locus of b' is drawn at a distance of 60 mm above the XY line and parallel to it. Locus of b is drawn at a distance of 50 mm below XY line and parallel to it.
2. Assume that the line is kept inclined to HP and parallel to VP. Draw the front view a' b' by considering a' as centre and true length 85 mm as radius, cut an arc in the locus of $b$ ' to mark $b_{1}^{\prime}$ with XY is the inclination of the line with $\operatorname{HP}(\theta)$.
3. Assume that the line is kept inclined to VP and parallel to HP. Draw the top view $\mathrm{ab}_{2}$ by considering a as centre and true length 85 mm as radius, cut an arc in the locus of $b$ to mark $b_{2}$. The front view length $a^{\prime} b_{2}^{\prime}$ is projected and obtained parallel to XY line. The inclination of top view $\mathrm{ab}_{2}$ with XY is the inclination of the line with $\operatorname{VP}(\phi)$.
4. Rotate the top view $a b_{1}$ to the required position by taking $a$ as centre, $a b_{1}$ as radius to get the intersection point $b$ with the locus of $b$. Join $a$ and $b$ to complete the top view $a b$ of the line. Rotate the front view $a^{\prime} b_{2}^{\prime}$ by taking $a^{\prime}$ as centre, $a^{\prime} b_{2}^{\prime}$ as radius to get
the intersection point $b^{\prime}$ with the locus of $b^{\prime}$. Join $a^{\prime}$ and $b^{\prime}$ to get the front view $a^{\prime} b^{\prime}$ of the line.
5. Check the result obtained by drawing the projector joining $b^{\prime}$ and $b$ which should be a vertical line.


EXAMPLE

A line AB has its end A 20 mm above HP and 25 mm in front of VP. The other end B is 45 mm above $H P$ and 55 mm in front of VP. The distance between the end projectors is 60 mm . Draw its projections. Also find the true length and true inclinations of the line with HP and VP.

## SOLUTION

Mark the projections of the end A. Its front view a' is 20 mm above XY and top view a is 25 mm below the XY line.

1. Draw the projector for the other end $B$ at a distance of 60 mm from $\mathrm{a}-\mathrm{a}$ '. Mark the projection of end $B$, its front view $b^{\prime}$ is 45 mm above $X Y$ line and top view $b$ is 55 mm below XY line. Join $a$ ' and $b$ ' to get front view $a$ ' $b$ ' of the line. Join $a$ and $b$ to get the top view ab of the line.
2. Draw the locus of the end $B$ in front and top views. Locus of $b^{\prime}$ is drawn passing through $b$ ' and parallel to XY line. Locus of $b$ is drawn passing through $b$ and parallel to XY line.
3. Rotate top view ab in the reverse order, take a as centre, top view length ab as radius, draw an arc to get $b_{1}$, on a line drawn parallel to XY line. Project $b_{1}$ to locus of $b$ ' to get $b_{1}$. Join $a^{\prime} b_{1}^{\prime}$ which has true length (TL) of the line. The inclination of $a^{\prime} b^{\prime}{ }_{1}$ with XY line is the true inclination of the with $\operatorname{HP}(\theta)$.
4. Rotate the front view $a^{\prime} b^{\prime}$ in the reverse order, take $a^{\prime}$ as centre, front view length a'b' as radius, draw an arc to get $b_{2}$, on a line drawn parallel to XY line. Project $b_{2}$ to the locus of $b$ to get $b_{2}$. Join $a b_{2}$ which has true length (TL) of the line. The inclination of $\mathrm{ab}_{2}$ with XY line is the inclination of the line with $\mathrm{VP}(\phi)$.


## EXAMPLE

The end $A$ of a line $A B$ is 10 mm in front of $V P$ and 20 mm above $H P$. The line is inclined at $30^{\circ}$ to $H P$ and front view is $45^{\circ}$ with XY. Top view is 60 mm long. Draw the projections. Find the true length and inclination with VP. Locate the traces.

## SOLUTION

Mark the projections of end A. Its front view a' 20 mm above XY and top view a is 10 mm below XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. Draw its top view $\mathrm{ab}_{1}$ from a which is parallel to the XY line for a length of 60 mm . Draw a vertical line (projector) from $b_{1}$ and draw a line from $a^{\prime}$ inclined at $30^{\circ}$ to XY line, intersecting at $\mathrm{b}_{1}$. The front view length a ' $\mathrm{b}_{1}$ is the true length (TL) of the line.
2. Draw the locus of $b^{\prime}$ passing through $b_{1}^{\prime}$ and parallel to $X Y$ line. Draw the front view $a^{\prime} b^{\prime}$ of the line inclined at $45^{\circ}$ to the XY line from a' and intersecting the locus of b ' at b'.
3. Draw the vertical line (projector) passing through b'. Rotate the top view ab by taking $a$ as centre and $a b_{1}$ as radius to intersect with the projector $a t b$. Draw the locus of $b$ passing through $b$ and parallel to XY line.
4. Rotate the front view a'b' in the reverse order. Take a' as centre, front view length $a^{\prime} b^{\prime}$ as radius and draw an arc to get $b_{2}$, parallel to XY line. Project $b_{2}$ to the locus of $b$ to get $b_{2}$. Join $a b_{2}$ which ahs the true length (TL) of the line. The inclination of $a b_{2}$ with the XY line is the true inclination of the line with $\mathrm{VP}(\phi)$.


## ANSWERS <br> $\phi=45^{\circ}$ <br> $\mathrm{TL}=70 \mathrm{~mm}$

## To mark the traces

1. Extend the front view a'b' to get the intersection point h' with XY line.
2. Produce the top view ab to get the intersection point $v$ with XY line.
3. Draw a vertical line from h' to intersect with the top view to get horizontal trace (HT). Draw another vertical line from $v$ to intersect with the front view to get the vertical trace (VT).

## EXAMPLE

A line $A B$ measuring 75 mm long has one of its ends 50 mm in front of $V P$ and 15 mm above HP. The top view of the line is 50 mm long. Draw and measure the front view. The other end is 15 mm in front of VP and is above HP. Determine the true inclinations and traces.


Mark the projections of end A, its front view a' is 15 mm above XY and top view a is 50 mm below XY line. Draw the locus of b , at a distance 15 mm below the XY line.

1. Assume that the line is kept inclined to HP and parallel to VP. The top view $a b_{1}$ is marked parallel to XY line for a length of 50 mm . Draw a vertical line (projector) from $\mathrm{b}_{1}$. Using true length 75 mm as radius and $\mathrm{a}^{\prime}$ as centre, draw an arc to intersect with the vertical line through $b_{1}$ to get $b_{1}^{\prime}$. Join a' and $b_{1}^{\prime}$ which represents the true length of the line. The inclination of $a^{\prime} b_{1}^{\prime}$ with $X Y$ is the inclination of the line with HP( $\theta$ ).
2. Assume that the line is kept inclined to VP and parallel to HP . Mark the top view $\mathrm{ab}_{2}$, by considering a as centre and true length 75 mm as radius, cut an arc on the locus of $b$ to get $b_{2}$. The inclination of $\mathrm{ab}_{2}$ with XY is the inclination of the line with $\operatorname{VP}(\phi)$. The front view length $a^{\prime} b_{2}$ is projected and obtained parallel to XY line.
3. Draw the locus of $b^{\prime}$ which is a line passing through $b_{1}^{\prime}$ and parallel to $X Y$ line.
4. Rotate the top view $\mathrm{ab}_{1}$ and front view $\mathrm{a}^{\prime} \mathrm{b}_{2}$ to the required position. Take a as centre, top view length $\mathrm{ab}_{1}$ as radius, draw an arc to intersect with the locus of $b$ at $b$. Join and $b$ to get the top view $a b$ of the line. Take $a^{\prime}$ as centre, front view length $a^{\prime} b_{2}$ as radius draw an arc to intersect with the locus of $b^{\prime}$ at $b^{\prime}$. Join $a^{\prime}$ and $b^{\prime}$ to get thefront view a'b' of the line.
5. Extend the front view a'b' to get the intersection point $h$ ' with XY line.
6. Produces the top view ab to get the intersection point $v$ with $X Y$ line.
7. Draw the vertical line from h' to intersect the top view to get the horizontal trace (HT) of the line.
8. Draw another vertical line from $v$ to intersect the front view to get vertical trace (VT) of the line.

## EXAMPLE

A line $A B$ is inclined at $30^{\circ}$ to VP has its ends 20 mm and 50 mm above the $H P$. The length of the front view is 65 mm and its VT is 10 mm above HP. Determine the true length of AB, its inclination with HP and its HT.

## SOLUTION

Mark the projection of end A, its front view a' is 20 mm above the XY line.

1. Draw the locus of b ' at a distance 50 mm above the $X Y$ line. Mark the front view a'b', by considering a' as centre and 65 mm length as radius, cut an arc on the locus of b' to get b'. Join a' with b' to get the front view a'b' of the line. Draw the locus of VT at a distance 10 mm above the XY line. Extend the front view a'b' to mark the VT.
2. Considering the length of the upto the VT where it meets VP. Project the top view of VT which is obtained on XY line and is marked as $v$. Assume that this line is inclined to VP and parallel to HP. From $v$, draw its top inclined at $30^{\circ}$ to XY line. Rotate its front view VT b' with VT as centre, parallel to XY line to get $b_{2}^{\prime}$ and project it to get $\mathrm{b}_{2}$ on the inclined line.


> ANSWERS
> $\theta=24^{\circ}$
> $T L=75 \mathrm{~mm}$
3. Draw the locus of $b$ passing through $b_{2}$ and parallel to $X Y$ line. Project $b$ ' to get $b$ on the locus of $b$. Join $v$ and $b$ to get the top view $v b$ for the line upto VT. Project $a^{\prime}$ to get the top view $a b$ of the required line.
4. Assume that the line is kept inclined to HP and parallel to VP. Rotate the top view ab by taking a as centre and $a b$ as radius to get parallel to XY line. Project $\mathrm{b}_{1}$ to locus of $b^{\prime}$ to get $b_{1}^{\prime}$. Join $a^{\prime}$ and $b_{1}^{\prime}$ to get the true length of the line. The inclination of $a^{\prime} b_{1}^{\prime}$ with XY is the inclination of the line with $\operatorname{HP}(\theta)$.

